

MICRO STRONGLY α^* -GENERALIZED HOMEOMORPHISM IN MICRO TOPOLOGICAL SPACES

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Abstract: Objective: S.Chandrasekar introduced Micro Topology in 2019. We created a new class of Micro Generalized Closed Sets in Micro Topological Spaces. In this paper, we introduced some forms of continuous functions namely strongly α^*g_M -homeomorphism. We study the connections of these functions with other forms of Micro continuous function. Also we established the various properties of these functions. Method: The study employs rigorous analysis of these Micro functions and formal proof technique to establish their properties. Comparative analysis with some existing function is conducted to highlight the distinct characteristics of these functions. Finding: The research identifies and confirms the unique features of strongly α^*g_M -homeomorphism under various operations, supported by detailed proofs and examples. Novelty: The research contributes to the mathematical literature by defining and exploring a new category of Micro function and comparing it with some of the existing functions.

Keywords: α^*g_M -closed, strongly α^*g_M -homeomorphism, strongly α^*g_M -closed map, strongly α^*g_M -open map and α^*g_M -open.

I. INTRODUCTION

Topology comprises one of the oldest and most significant topics of Mathematics; the idea of Topology is becoming vaster and more complex by the day. Several versions of Topologies have recently been developed until now. The concept of Nano Topology was introduced by Lellis Thivagar in 2013 which is defined in terms of the lower approximation, higher approximation, and the boundary area, subset of the universe set involving the equivalence relation. The notation of approximation and the boundary area of a set were originally introduced by Pawlak in order to introduce the concept of rough sets theory. Micro Topology was an expansion of Nano Topology. S.Chandrasekar introduced and researched Micro Semi open, Micro Pre open, Micro Semi continuous and Micro Pre continuous in Micro Topological Spaces. S. Chandrasekar and G. Swathi introduced by Micro α -open sets, Micro α -closure, & Micro α -interior in Micro Topological Spaces. Ibrahim.H.Z. introduced Micro β -open and Micro g -Closed Sets in Micro Topological Spaces. Recently, R.Anandhi and N. Balamani invented the idea of Micro α -Generalised Closed Sets on Micro Topological Spaces and investigated the corresponding Micro separation axioms. A New Class of Micro Generalized Closed Sets in Micro Topological Spaces, that is, α^*g_M -Closed Sets is introduced by P.Anbarasi Rodrigo and K.Selvakumari Prasha. The purpose of this paper was too introduced and analyzed the concepts of as strongly α^*g_M -homeomorphism, strongly α^*g_M -closed map and strongly α^*g_M -open map in Micro Topological Spaces. Some of the basic properties of strongly α^*g_M -homeomorphism, strongly α^*g_M -closed map and strongly α^*g_M -open map are analyzed.

II. METHODOLOGY

This paper introduces and extends the concepts of strongly α^*g_M -homeomorphism within the frameworks of Micro Topological Spaces. It explores the connection between the newly defined functions and the previously established Micro continuous through the application of definitions. Additionally, the paper provides examples to demonstrate the converse parts of these relationships and other properties.

III. PRELIMINARIES

In this portion, we review a few essential Micro Topological Spaces definitions and outcomes that will be important in the sequel.

Definition: 3.1 Let $U \neq \phi$, finite class of elements known as universe & $\mathcal{R}$ denotes the equivalence relation in U termed as indiscernibility relation. Therefore U is parted into distinct equivalence classes. Object from same equivalence class has been named as indiscernible with one another. And $(U, \mathcal{R})$, the pair is approximation space. Take $X \subseteq_M U$.

- 1) The lower Approximation of X in regards to $\mathcal{R}$ would be the class of entire elements, that could be for certain classified as X in terms of $\mathcal{R}$ which is notated as $\mathbb{L}_{\mathcal{R}}(X)$. i.e. $\mathbb{L}_{\mathcal{R}}(X) = \bigcup_{X \in U} \{ \mathcal{R}(X) : \mathcal{R}(X) \subseteq_M X \}$, here $\mathcal{R}(X)$ refer the equivalence class gained by $X \in U$.
- 2) The upper Approximation of X under $\mathcal{R}$ would be the class of elements, which could be possibly characterized as X by $\mathcal{R}$ & notated by $\mathbb{U}_{\mathcal{R}}(X)$. i.e. $\mathbb{U}_{\mathcal{R}}(X) = \bigcup_{X \in U} \{ \mathcal{R}(X) : \mathcal{R}(X) \cap X \neq \phi \}$.
- 3) The boundary area of X in regards to $\mathcal{R}$ is the class of elements, that could be specified neither as X nor as not X in regards with $\mathcal{R}$ & notated by $\mathbb{B}_{\mathcal{R}}(X)$. i.e. $\mathbb{B}_{\mathcal{R}}(X) = \mathbb{U}_{\mathcal{R}}(X) - \mathbb{L}_{\mathcal{R}}(X)$.

Definition: 3.2 Consider universe $(U, \mathcal{R})$, equivalence relation in U & $\tau_{\mathcal{R}}(X) = \{U, \phi, \mathbb{L}_{\mathcal{R}}(X), \mathbb{U}_{\mathcal{R}}(X), \mathbb{B}_{\mathcal{R}}(X)\}$, here $X \subseteq_M U$. So that $\tau_{\mathcal{R}}(X)$ fulfills the below criteria:

- 1) $U, \phi \in \tau_{\mathcal{R}}(X)$.
- 2) The union of objects from the sub-classes of $\tau_{\mathcal{R}}(X)$ lies in $\tau_{\mathcal{R}}(X)$.
- 3) The intersection of object of any finite sub-classes of $\tau_{\mathcal{R}}(X)$ lies in $\tau_{\mathcal{R}}(X)$. i.e. $\tau_{\mathcal{R}}(X)$ is a

Topology in U said as Nano Topology on U in regards to X . $(U, \tau_{\mathcal{R}}(X))$ termed as Nano Topological Space. Object from $\tau_{\mathcal{R}}(X)$ are the Nano open sets and its complements is Nano closed sets.

Definition: 3.3 [7] Consider $(U, \tau_{\mathcal{R}}(X))$, an Nano Topological Spaces. So $\mu_{\mathcal{R}}(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_{\mathcal{R}}(X)\}$ & it is referred as Micro Topology of $\tau_{\mathcal{R}}(X)$ and $\mu \notin \tau_{\mathcal{R}}$ meet the given axioms:

- 1) $U, \phi \in \mu_{\mathcal{R}}(X)$.
- 2) The union of objects from the Sub-classes of $\mu_{\mathcal{R}}(X)$ lies in $\mu_{\mathcal{R}}(X)$.
- 3) The intersection of objects from any finite sub-classes of $\mu_{\mathcal{R}}(X)$ lies in $\mu_{\mathcal{R}}(X)$. So $\mu_{\mathcal{R}}(X)$

is said as the Micro Topological on U in regards to X . $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ is termed as Micro Topological Spaces and objects from $\mu_{\mathcal{R}}(X)$ are the Micro open & its complement is termed as a Micro closed sets.

Definition: 3.4 [1] Consider $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ a Micro Topological Spaces. A subset A of U is said to be α^*g_M -closed set if $\mathcal{M}\alpha^*cl(A) \subseteq_M int U$, whence $A \subseteq_M U$ & U is a Micro α -open in U .

Example: 3.5 Take $U = \{a, b, c, d\} \ni U/\mathcal{R} = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{b, c\} \subseteq_M U$. So $\tau_{\mathcal{R}}(X) = \{U, \phi, \{b, d\}\}$, $\mu = \{a\} \notin \tau_{\mathcal{R}}(X)$ and $\mu_{\mathcal{R}}(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Then α^*g_M -closed = $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$.

Result: 3.6 [1] Every $\mathcal{M}$ -closed is α^*g_M -closed.

Definition: 3.7 [4] A Micro Topological Spaces $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ is said to be $\alpha^*g_M\mathcal{T}_{1/2}$ -space or α^*g_M -space if every α^*g_M -closed in U is Micro closed in U .

Definition: 3.8 [2] Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. A function $F: U \rightarrow V$ is called α^*g_M -continuous functions if the inverse image of every α^*g_M -closed in U is Micro closed in V .

Result: 3.9 [4]

- 1) $\alpha^*g_{\mathcal{M}}\text{-int}(\mathcal{S})=\mathcal{S}$ is a $\alpha^*g_{\mathcal{M}}$ -open in $\mathcal{U}$.
- 2) $\alpha^*g_{\mathcal{M}}\text{-cl}(\mathcal{S})=\mathcal{S}$ is a $\alpha^*g_{\mathcal{M}}$ -closed in $\mathcal{U}$.

Result: 3.10 [2] Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is $\alpha^*g_{\mathcal{M}}$ -continuous functions if and only if the each of the following holds.

- 1) $\alpha^*g_{\mathcal{M}}\text{-cl}(\mathbb{F}^{-1}(\mathcal{S})) \subseteq_{\mathcal{M}} \mathbb{F}^{-1}(\mathcal{M}\text{cl}(\mathcal{S})), \forall \mathcal{S} \subseteq_{\mathcal{M}} \mathcal{V}$.
- 2) $\mathbb{F}(\alpha^*g_{\mathcal{M}}\text{-cl}(\mathcal{S})) \subseteq_{\mathcal{M}} \mathcal{M}\text{cl}(\mathbb{F}(\mathcal{S})), \forall \mathcal{S} \subseteq_{\mathcal{M}} \mathcal{U}$.
- 3) $\mathbb{F}^{-1}(\mathcal{M}\text{int}(\mathcal{T})) \subseteq_{\mathcal{M}} \alpha^*g_{\mathcal{M}}\text{-int}(\mathbb{F}^{-1}(\mathcal{T})), \forall \mathcal{T} \subseteq_{\mathcal{M}} \mathcal{V}$.
- 4) $\mathbb{F}(\alpha^*g_{\mathcal{M}}\text{-int}(\mathcal{T})) \subseteq_{\mathcal{M}} \mathcal{M}\text{-int}(\mathbb{F}(\mathcal{T})), \forall \mathcal{T} \subseteq_{\mathcal{M}} \mathcal{U}$.

Definition: 3.11 [2] Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called $\alpha^*g_{\mathcal{M}}$ -irresolute function if $\mathbb{F}^{-1}(\mathcal{S})$ is $\alpha^*g_{\mathcal{M}}$ -closed in $\mathcal{U} \forall \alpha^*g_{\mathcal{M}}$ -closed in $\mathcal{V}$.

Definition: 3.12 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called Micro closed map if the image of every Micro closed set $\mathcal{S}$ in $\mathcal{U}$ is Micro closed in $\mathcal{V}$.

Definition: 3.13 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called Micro open map if the image of every Micro open set $\mathcal{S}$ in $\mathcal{U}$ is Micro open in $\mathcal{V}$.

Definition: 3.14 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A bijective map $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called strongly Micro homeomorphism if both $\mathbb{F}$ Micro irresolute and strongly Micro closed map.

Definition: 3.15 [6] Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called $\alpha^*g_{\mathcal{M}}$ -closed map if the image of every Micro closed set $\mathcal{S}$ in $\mathcal{U}$ is $\alpha^*g_{\mathcal{M}}$ -closed in $\mathcal{V}$.

Definition: 3.16 [6] Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called $\alpha^*g_{\mathcal{M}}$ -open map if the image of every Micro open set $\mathcal{S}$ in $\mathcal{U}$ is $\alpha^*g_{\mathcal{M}}$ -open in $\mathcal{V}$.

Definition: 3.17 [6] Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called $\alpha^*g_{\mathcal{M}}$ -homeomorphism if

- 1) $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}$ -continuous
- 2) $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}$ -closed map
- 3) $\mathbb{F}$ is bijection.

Definition: 3.18 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called strongly Micro closed map if for each Micro closed set $\mathcal{S}$ in $\mathcal{U}$, $\mathbb{F}(\mathcal{S})$ is Micro closed in $\mathcal{V}$.

Definition: 3.19 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called strongly Micro open map if for each Micro open set $\mathcal{S}$ in $\mathcal{U}$, $\mathbb{F}(\mathcal{S})$ is Micro open in $\mathcal{V}$.

IV. STRONGLY $\alpha^*g_{\mathcal{M}}$ -CLOSED MAP AND STRONGLY $\alpha^*g_{\mathcal{M}}$ -OPEN MAP

Definition: 4.1 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called strongly $\alpha^*g_{\mathcal{M}}$ -closed map if for each $\alpha^*g_{\mathcal{M}}$ -closed set $\mathcal{S}$ in $\mathcal{U}$, $\mathbb{F}(\mathcal{S})$ is $\alpha^*g_{\mathcal{M}}$ -closed in $\mathcal{V}$.

Instance: 4.2 Take $\mathcal{U} = \{a, b, c\} \ni \mathcal{U}/\mathcal{R} = \{\{a\}, \{b, c\}\}$ and $\mathcal{X} = \{b\} \subseteq_{\mathcal{M}} \mathcal{U}$. So $\tau_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U}, \phi, \{b, c\}\}$, $\mu = \{a, b\} \notin \tau_{\mathcal{R}}(\mathcal{X})$ and $\mu_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U}, \phi, \{b\}, \{a, b\}, \{b, c\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-closed} = \{\mathcal{U}, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$. Let $\mathcal{V} = \{a, b, c\}$ with $\mathcal{V}/\mathcal{R} = \{\{b\}, \{a\}, \{c\}\}$ and $\mathcal{M} = \{b, c\} \subseteq_{\mathcal{M}} \mathcal{V}$. Then $\tau_{\mathcal{R}}^*(\mathcal{M}) = \{\mathcal{V}, \phi, \{b, c\}\}$, $\mu^* = \{a\} \notin \tau_{\mathcal{R}}^*(\mathcal{M})$ and $\mu_{\mathcal{R}}^*(\mathcal{M}) = \{\mathcal{V}, \phi, \{a\}, \{b, c\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-closed} = \{\mathcal{V}, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$. Let $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ be identity function defined by $\mathbb{F}(a)=a, \mathbb{F}(b)=b, \mathbb{F}(c)=c$. Hence $\mathbb{F}$ is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

Definition: 4.3 Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}), \mu_{\mathcal{R}}(\mathcal{X}))$ and $(\mathcal{V}, \tau_{\mathcal{R}}^*(\mathcal{M}), \mu_{\mathcal{R}}^*(\mathcal{M}))$ be two Micro Topological Spaces. A function $\mathbb{F}: \mathcal{U} \rightarrow \mathcal{V}$ is called strongly $\alpha^*g_{\mathcal{M}}$ -open map if for each $\alpha^*g_{\mathcal{M}}$ -open set $\mathcal{S}$ in $\mathcal{U}$, $\mathbb{F}(\mathcal{S})$ is $\alpha^*g_{\mathcal{M}}$ -open in $\mathcal{V}$.

Instance: 4.4 Take $\mathbb{U}=\{a,b,c\} \ni \mathbb{U}/\mathcal{R}=\{\{a\},\{b,c\}\}$ and $\mathbb{X}=\{b\} \subseteq_{\mathcal{M}} \mathbb{U}$. So $\tau_{\mathcal{R}}(\mathbb{X})=\{\mathbb{U},\phi,\{b,c\}\}$, $\mu=\{a,b\} \notin \tau_{\mathcal{R}}(\mathbb{X})$ and $\mu_{\mathcal{R}}(\mathbb{X})=\{\mathbb{U},\phi,\{b\},\{a,b\},\{b,c\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-open}=\{\mathbb{U},\phi,\{a\},\{b\},\{c\},\{a,c\},\{a,b\},\{b,c\}\}$. Let $\mathbb{V}=\{a,b,c\}$ with $\mathbb{V}/\mathcal{R}=\{\{b\},\{a\},\{c\}\}$ and $\mathbb{M}=\{b,c\} \subseteq_{\mathcal{M}} \mathbb{V}$. Then $\tau^*_{\mathcal{R}}(\mathbb{M})=\{\mathbb{V},\phi,\{b,c\}\}$, $\mu^*=\{a\} \notin \tau^*_{\mathcal{R}}(\mathbb{M})$ and $\mu^*_{\mathcal{R}}(\mathbb{M})=\{\mathbb{V},\phi,\{a\},\{b,c\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-open}=\{\mathbb{V},\phi,\{a\},\{b\},\{c\},\{a,c\},\{a,b\},\{b,c\}\}$. Let $\mathbb{F}:\mathbb{U} \rightarrow \mathbb{V}$ be identity function defined by $\mathbb{F}(a)=a$, $\mathbb{F}(b)=b$, $\mathbb{F}(c)=c$. Hence $\mathbb{F}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-open}$ map.

Theorem: 4.5 Every strongly $\alpha^*g_{\mathcal{M}}\text{-closed(open)}$ map is $\alpha^*g_{\mathcal{M}}\text{-closed(open)}$ map.

Proof: Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$ and $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ be two Micro Topological Spaces. Let $\mathbb{S}$ be a Micro closed in $\mathbb{U}$. Since by result 3.6, $\mathbb{S}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{U}$. Since $\mathbb{F}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Hence $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Remark: 4.6 Vice versa of mentioned theorem is not true.

Instance: 4.7 Take $\mathbb{U}=\{a,b,c,d\} \ni \mathbb{U}/\mathcal{R}=\{\{a\},\{c\},\{b,d\}\}$ and $\mathbb{X}=\{b,c\} \subseteq_{\mathcal{M}} \mathbb{U}$. So $\tau_{\mathcal{R}}(\mathbb{X})=\{\mathbb{U},\phi,\{b,d\}\}$, $\mu=\{a\} \notin \tau_{\mathcal{R}}(\mathbb{X})$ and $\mu_{\mathcal{R}}(\mathbb{X})=\{\mathbb{U},\phi,\{a\},\{b,d\},\{a,b,d\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-closed}=\{\mathbb{U},\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\}\}$ and $\mathcal{M}\text{-C}=\{\mathbb{U},\phi,\{c\},\{a,c\},\{b,c,d\}\}$. Let $\mathbb{V}=\{a,b,c,d\} \ni \mathbb{V}/\mathcal{R}=\{\{a\},\{c\},\{b,d\}\}$ and $\mathbb{M}=\{c,d\} \subseteq_{\mathcal{M}} \mathbb{V}$. So that $\tau^*_{\mathcal{R}}(\mathbb{M})=\{\mathbb{V},\phi,\{c\},\{b,d\},\{b,c,d\}\}$, $\mu^*=\{a\} \notin \tau^*_{\mathcal{R}}(\mathbb{M})$ and $\mu^*_{\mathcal{R}}(\mathbb{M})=\{\mathbb{V},\phi,\{b\},\{c\},\{b,d\},\{a,b\},\{b,c\},\{a,b,c\},\{b,c,d\},\{a,b,d\}\}$. Then $\alpha^*g_{\mathcal{M}}\text{-closed}=\{\mathbb{V},\phi,\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\}\}$. Let $\mathbb{F}:\mathbb{U} \rightarrow \mathbb{V}$ be defined by $\mathbb{F}(a)=c$, $\mathbb{F}(b)=b$, $\mathbb{F}(c)=a$, $\mathbb{F}(d)=b$. Thus $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map but not strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map since $\mathbb{F}(\{b,d\})=\{b,d\}$ is not $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$.

Theorem: 4.8 The composition of two strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Proof: Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$ and $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ be two Micro Topological Spaces. Let $\mathbb{S}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{U}$. Since $\mathbb{F}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Since $\mathbb{G}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, $\mathbb{G}(\mathbb{F}(\mathbb{S}))$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{W}$. Hence $\mathbb{G} \circ \mathbb{F}:\mathbb{U} \rightarrow \mathbb{W}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Theorem: 4.9 Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$, $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ and $(\mathbb{W},\tau^{**}_{\mathcal{R}}(\mathbb{N}),\mu^{**}_{\mathcal{R}}(\mathbb{N}))$ be three Micro Topological Spaces. Let $\mathbb{F}:\mathbb{U} \rightarrow \mathbb{V}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ map and $\mathbb{G}:\mathbb{V} \rightarrow \mathbb{W}$ be a strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map. Then $\mathbb{G} \circ \mathbb{F}:\mathbb{U} \rightarrow \mathbb{W}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Proof: Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$, $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ and $(\mathbb{W},\tau^{**}_{\mathcal{R}}(\mathbb{N}),\mu^{**}_{\mathcal{R}}(\mathbb{N}))$ be three Micro Topological Spaces. Let $\mathbb{S}$ be a Micro closed in $\mathbb{U}$. Since $\mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, then $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Since $\mathbb{G}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, then $\mathbb{G}(\mathbb{F}(\mathbb{S}))$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{W}$. Hence $\mathbb{G} \circ \mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Theorem: 4.10 Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$, $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ and $(\mathbb{W},\tau^{**}_{\mathcal{R}}(\mathbb{N}),\mu^{**}_{\mathcal{R}}(\mathbb{N}))$ be three Micro Topological Spaces. Let $\mathbb{F}:\mathbb{U} \rightarrow \mathbb{V}$ be a strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, $\mathbb{G}:\mathbb{V} \rightarrow \mathbb{W}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ map and $\mathbb{V}$ be a $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space. Then $\mathbb{G} \circ \mathbb{F}:\mathbb{U} \rightarrow \mathbb{W}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Proof: Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$, $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ and $(\mathbb{W},\tau^{**}_{\mathcal{R}}(\mathbb{N}),\mu^{**}_{\mathcal{R}}(\mathbb{N}))$ be three Micro Topological Spaces. Let $\mathbb{S}$ be a Micro closed in $\mathbb{U}$. Then $\mathbb{S}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{U}$. Since $\mathbb{F}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, then $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Since $\mathbb{V}$ is $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space, $\mathbb{F}(\mathbb{S})$ is Micro closed in $\mathbb{V}$. Since $\mathbb{G}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, then $\mathbb{G}(\mathbb{F}(\mathbb{S}))$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{W}$. Hence $\mathbb{G} \circ \mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Theorem: 4.11 Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$, $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ and $(\mathbb{W},\tau^{**}_{\mathcal{R}}(\mathbb{N}),\mu^{**}_{\mathcal{R}}(\mathbb{N}))$ be three Micro Topological Spaces. Let $\mathbb{U}$ be $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space. If $\mathbb{F}:\mathbb{U} \rightarrow \mathbb{V}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ map and $\mathbb{G}:\mathbb{V} \rightarrow \mathbb{W}$ be a strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, then $\mathbb{G} \circ \mathbb{F}:\mathbb{U} \rightarrow \mathbb{W}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Proof: Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$, $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ and $(\mathbb{W},\tau^{**}_{\mathcal{R}}(\mathbb{N}),\mu^{**}_{\mathcal{R}}(\mathbb{N}))$ be three Micro Topological Spaces. Let $\mathbb{S}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{U}$. Since $\mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\mathcal{T}_{1/2}$ -space, then $\mathbb{S}$ is Micro closed in $\mathbb{U}$. Since $\mathbb{F}$ is a $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Since $\mathbb{G}$ is strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map, then $\mathbb{G}(\mathbb{F}(\mathbb{S}))$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{W}$. Hence $\mathbb{G} \circ \mathbb{F}$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Theorem: 4.12 Let $(\mathbb{U},\tau_{\mathcal{R}}(\mathbb{X}),\mu_{\mathcal{R}}(\mathbb{X}))$ and $(\mathbb{V},\tau^*_{\mathcal{R}}(\mathbb{M}),\mu^*_{\mathcal{R}}(\mathbb{M}))$ be two Micro Topological Spaces. Let $\mathbb{F}:\mathbb{U} \rightarrow \mathbb{V}$ be a bijective function. Then $\mathbb{F}^{-1}:\mathbb{V} \rightarrow \mathbb{U}$ is $\alpha^*g_{\mathcal{M}}\text{-irresolute}$ if and only if $\mathbb{F}$ is a strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map.

Proof: Let $\mathbb{S}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{U}$. Since $\mathbb{F}^{-1}$ is $\alpha^*g_{\mathcal{M}}\text{-irresolute}$ and bijective function, $(\mathbb{F}^{-1})^{-1}(\mathbb{S})=\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Then $\mathbb{F}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Hence $\mathbb{F}$ is a strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map. **Conversely**, let $\mathbb{S}$ be a $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{U}$. Since $\mathbb{F}$ is a strongly $\alpha^*g_{\mathcal{M}}\text{-closed}$ map and bijective function, $\mathbb{F}(\mathbb{S})=(\mathbb{F}^{-1})^{-1}(\mathbb{S})$ is $\alpha^*g_{\mathcal{M}}\text{-closed}$ in $\mathbb{V}$. Hence $\mathbb{F}^{-1}$ is a $\alpha^*g_{\mathcal{M}}\text{-irresolute}$.

Theorem: 4.13 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let $F: U \rightarrow V$ be a strongly $\alpha^*g_{\mathcal{M}}$ -closed map if and only if $\alpha^*g_{\mathcal{M}}\text{-cl}(F(S)) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(S)), \forall S \subseteq_{\mathcal{M}} U$.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let F be a strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Let S be $\alpha^*g_{\mathcal{M}}$ -closed in U . Then $\mathcal{M}\text{-cl}(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in U . Since F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map, $F(\mathcal{M}\text{-cl}(S))$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Since $S \subseteq_{\mathcal{M}} \mathcal{M}\text{-cl}(S)$, $F(S) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(S))$. Then $\alpha^*g_{\mathcal{M}}\text{-cl}(F(S)) \subseteq_{\mathcal{M}} \alpha^*g_{\mathcal{M}}\text{-cl}(F(\mathcal{M}\text{-cl}(S)))$. Thus $\alpha^*g_{\mathcal{M}}\text{-cl}(F(S)) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(S))$. **Conversely**, let $\alpha^*g_{\mathcal{M}}\text{-cl}(F(S)) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(S))$. Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in U . Then $\alpha^*g_{\mathcal{M}}\text{-cl}(S) = S$. Then $F(S) = F(\alpha^*g_{\mathcal{M}}\text{-cl}(S))$. By hypothesis, $\alpha^*g_{\mathcal{M}}\text{-cl}(F(S)) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(S)) = F(S)$ which implies $\alpha^*g_{\mathcal{M}}\text{-cl}(F(S)) \subseteq_{\mathcal{M}} F(S)$. But $F(S) \subseteq_{\mathcal{M}} F(\alpha^*g_{\mathcal{M}}\text{-cl}(S))$ is always true. Then $F(S) = F(\alpha^*g_{\mathcal{M}}\text{-cl}(S))$. Thus $F(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Hence F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

Theorem: 4.14 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let $F: U \rightarrow V$ be a strongly $\alpha^*g_{\mathcal{M}}$ -closed map and bijection. Then $F^{-1}(\alpha^*g_{\mathcal{M}}\text{-cl}(S)) \subseteq_{\mathcal{M}} \mathcal{M}\text{-cl}(F^{-1}(S)), \forall S \subseteq_{\mathcal{M}} V$.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let F be a strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Let S be $\alpha^*g_{\mathcal{M}}$ -closed in U . Since F is bijection, F^{-1} is bijection. Since F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map and bijection, then $F^{-1}(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in U . By theorem 4.14, $\alpha^*g_{\mathcal{M}}\text{-cl}(F(F^{-1}(S))) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(F^{-1}(S)))$ which implies $\alpha^*g_{\mathcal{M}}\text{-cl}(S) \subseteq_{\mathcal{M}} F(\mathcal{M}\text{-cl}(F^{-1}(S)))$ and hence $F^{-1}(\alpha^*g_{\mathcal{M}}\text{-cl}(S)) \subseteq_{\mathcal{M}} \mathcal{M}\text{-cl}(F^{-1}(S)), \forall S \subseteq_{\mathcal{M}} V$.

Theorem: 4.15 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$, $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ and $(W, \tau_{\mathcal{R}}^{**}(N), \mu_{\mathcal{R}}^{**}(N))$ be three Micro Topological Spaces. Let $F: U \rightarrow V$ and $G: V \rightarrow W$ be a two functions such that $G \circ F: U \rightarrow W$ is strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Then the following statements are holds.

- 1) If G is $\alpha^*g_{\mathcal{M}}$ -irresolute and injective, then F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.
- 2) If F is $\alpha^*g_{\mathcal{M}}$ -irresolute and surjective, then G is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$, $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ and $(W, \tau_{\mathcal{R}}^{**}(N), \mu_{\mathcal{R}}^{**}(N))$ be three Micro Topological Spaces. Let $F: U \rightarrow V$ and $G: V \rightarrow W$ be a two functions such that $G \circ F: U \rightarrow W$ is $\alpha^*g_{\mathcal{M}}$ -closed.

- 1) Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in U . By hypothesis, $G \circ F$ is strongly $\alpha^*g_{\mathcal{M}}$ -closed map, then $G \circ F(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in W . Since G is $\alpha^*g_{\mathcal{M}}$ -irresolute, $G^{-1}(G \circ F(S))$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Since G is injective, $F(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Hence F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.
- 2) Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in V . Since F is $\alpha^*g_{\mathcal{M}}$ -irresolute, $F^{-1}(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in U . As $G \circ F$ is strongly $\alpha^*g_{\mathcal{M}}$ -closed map and F is surjective, then $(G \circ F)F^{-1}(S) = G(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in W . Hence G is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

Theorem: 4.16 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let $F: U \rightarrow V$ be a bijective functions. Then the following statements are equivalent.

- 1) F is strongly $\alpha^*g_{\mathcal{M}}$ -open map.
- 2) F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.
- 3) $F^{-1}: V \rightarrow U$ is $\alpha^*g_{\mathcal{M}}$ -irresolute function.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces.

1) $\Rightarrow$ 2) Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in U . Then $U-S$ is a $\alpha^*g_{\mathcal{M}}$ -open in U . Since F is strongly $\alpha^*g_{\mathcal{M}}$ -open map, $F(U-S)$ is $\alpha^*g_{\mathcal{M}}$ -open in V . Since F is bijection function, $F(U-S) = V - F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -open in V . Then $F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -closed in V . Hence F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

2) $\Rightarrow$ 3) Let F be strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in U . Since F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map, $F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -closed in V . Since F is bijection function, $(F^{-1})^{-1}(S) = F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -closed in V . Hence F^{-1} is $\alpha^*g_{\mathcal{M}}$ -irresolute function.

3) $\Rightarrow$ 1) Let S $\alpha^*g_{\mathcal{M}}$ -open in U . Since F^{-1} is $\alpha^*g_{\mathcal{M}}$ -irresolute function, $(F^{-1})^{-1}(S)$ is $\alpha^*g_{\mathcal{M}}$ -open in V . Since F is bijection function, $(F^{-1})^{-1}(S) = F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -open in V . Hence F is strongly $\alpha^*g_{\mathcal{M}}$ -open map.

Theorem: 4.17 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. If F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map if and only if for each subset S of V and for each $\alpha^*g_{\mathcal{M}}$ -open set T of U containing $F^{-1}(S)$ there exists a $\alpha^*g_{\mathcal{M}}$ -open set R of V such that $S \subseteq_{\mathcal{M}} R$ and $F^{-1}(R) \subseteq_{\mathcal{M}} T$.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Suppose F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Let $S \subseteq_{\mathcal{M}} R$ and T is an $\alpha^*g_{\mathcal{M}}$ -open set of U . Then $U-T$ is $\alpha^*g_{\mathcal{M}}$ -closed in U . Since F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map, $F(U-T)$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . This implies $F(U-T) = V - F(T)$ is $\alpha^*g_{\mathcal{M}}$ -open in V . Since $F^{-1}(S) \subseteq_{\mathcal{M}} T$, we have $S \subseteq_{\mathcal{M}} R$ and $F^{-1}(R) \subseteq_{\mathcal{M}} T$. **Conversely**, Suppose S is $\alpha^*g_{\mathcal{M}}$ -closed in U . Then $F^{-1}(V-F(S)) \subseteq_{\mathcal{M}} U-S$ and $U-S$ is $\alpha^*g_{\mathcal{M}}$ -open in U . By assumption there exists a $\alpha^*g_{\mathcal{M}}$ -open set R of V such that $V-F(S) \subseteq_{\mathcal{M}} R$ and $F^{-1}(R) \subseteq_{\mathcal{M}} (U-S)$. Therefore, $S \subseteq_{\mathcal{M}} (U-F^{-1}(R))$. So $V-R \subseteq_{\mathcal{M}} F(S) \subseteq_{\mathcal{M}} U-F(F^{-1}(R)) \subseteq_{\mathcal{M}} V-R$ which implies that $F(S) = V-R \dots (1)$. Since $V-R$ is $\alpha^*g_{\mathcal{M}}$ -closed in V , $F(S)$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Therefore F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

V. STRONGLY $\alpha^*g_{\mathcal{M}}$ -HOMEOMORPHISM ($\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$)

Definition: 5.1 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. A bijective map $F: U \rightarrow V$ is called strongly $\alpha^*g_{\mathcal{M}}$ -homeomorphism (That is, $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$) if both F is $\alpha^*g_{\mathcal{M}}$ -irresolute and strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

The spaces $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ are $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$ if there exists a $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$ from $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ onto $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$.

Instance: 5.2 Take $U = \{a, b, c\} \ni U/\mathcal{R} = \{\{a\}, \{b, c\}\}$ and $X = \{b\} \subseteq_{\mathcal{M}} U$. So $\tau_{\mathcal{R}}(X) = \{U, \phi, \{b, c\}\}$, $\mu = \{a, b\} \notin \tau_{\mathcal{R}}(X)$ and $\mu_{\mathcal{R}}(X) = \{U, \phi, \{b\}, \{a, b\}, \{b, c\}\}$. Then $\alpha^*g_{\mathcal{M}}$ -closed = $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$. Let $V = \{a, b, c\}$ with $V/\mathcal{R} = \{\{b\}, \{a\}, \{c\}\}$ and $M = \{b, c\} \subseteq_{\mathcal{M}} V$. Then $\tau_{\mathcal{R}}^*(M) = \{V, \phi, \{b, c\}\}$, $\mu^* = \{a\} \notin \tau_{\mathcal{R}}^*(M)$ and $\mu_{\mathcal{R}}^*(M) = \{V, \phi, \{a\}, \{b, c\}\}$. Then $\alpha^*g_{\mathcal{M}}$ -closed = $\{V, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}\}$. Let $F: U \rightarrow V$ be identity function defined by $F(a) = a, F(b) = b, F(c) = c, F^{-1}(a) = a, F^{-1}(b) = b, F^{-1}(c) = c$. Thus F is $\alpha^*g_{\mathcal{M}}$ -irresolute and strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Hence F is $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$.

Theorem: 5.3 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let $F: U \rightarrow V$ and $G: V \rightarrow W$ be two $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$. Then $G \circ F: U \rightarrow W$ is $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in W . But $(G \circ F)^{-1}(S) = F^{-1}(G^{-1}(S)) = F^{-1}(T)$ where $G^{-1}(S) = T$. Since F and G are $\alpha^*g_{\mathcal{M}}$ -irresolute, $F^{-1}(T)$ is $\alpha^*g_{\mathcal{M}}$ -closed in U and $G^{-1}(S) = T$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Then $(G \circ F)^{-1}(S) = F^{-1}(T)$ is $\alpha^*g_{\mathcal{M}}$ -closed in V . Thus $G \circ F$ is $\alpha^*g_{\mathcal{M}}$ -irresolute. Also for a $\alpha^*g_{\mathcal{M}}$ -closed R in U , $G \circ F(P) = G(F(P)) = G(R)$ where $F(P) = R$. Since F is strongly $\alpha^*g_{\mathcal{M}}$ -closed, then $F(P) = R$ is $\alpha^*g_{\mathcal{M}}$ -closed in V and G is strongly $\alpha^*g_{\mathcal{M}}$ -closed, then $G(F(P)) = G(R)$ is $\alpha^*g_{\mathcal{M}}$ -closed in W . But $G \circ F(P) = G(R)$ is $\alpha^*g_{\mathcal{M}}$ -closed in W . This implies $G \circ F$ is strongly $\alpha^*g_{\mathcal{M}}$ -closed map. Hence $G \circ F$ is $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$.

Theorem: 5.4 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let $F: U \rightarrow V$ be a bijective map and $\alpha^*g_{\mathcal{M}}$ -irresolute maps. Then the following are equivalent.

- 1) F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.
- 2) F is $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$.
- 3) F is strongly $\alpha^*g_{\mathcal{M}}$ -open map.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces.

- 1) $\Rightarrow$ 2) By hypothesis, $F: U \rightarrow V$ be a bijective map, $\alpha^*g_{\mathcal{M}}$ -irresolute and $\alpha^*g_{\mathcal{M}}$ -closed map. Hence F is $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$.
- 2) $\Rightarrow$ 3) Let F be $\mathcal{S}_{\mathcal{M}}\alpha^*g_{\mathcal{M}}\mathcal{H}$. Then F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map and $\alpha^*g_{\mathcal{M}}$ -irresolute maps. Let S be a $\alpha^*g_{\mathcal{M}}$ -open in U . Then $U-S$ is a $\alpha^*g_{\mathcal{M}}$ -closed in U . Since F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map, $F(U-S)$ is a $\alpha^*g_{\mathcal{M}}$ -closed in V . That is $F(U-S) = V - F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -closed in V implies $F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -open in V . Hence F is strongly $\alpha^*g_{\mathcal{M}}$ -open map.
- 3) $\Rightarrow$ 1) Let S be a $\alpha^*g_{\mathcal{M}}$ -closed in U . Then $U-S$ is a $\alpha^*g_{\mathcal{M}}$ -open in U . By hypothesis, $F(U-S)$ is a $\alpha^*g_{\mathcal{M}}$ -open in V . That is $F(U-S) = V - F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -open in V implies $F(S)$ is a $\alpha^*g_{\mathcal{M}}$ -closed in V . Hence F is strongly $\alpha^*g_{\mathcal{M}}$ -closed map.

Theorem: 5.5 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Then F is $\mathcal{S}_M \alpha^* g_M \mathcal{H}$ if and only if F is $\alpha^* g_M$ -open map and $\alpha^* g_M$ -continuous.

Proof: Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let F be a $\mathcal{S}_M \alpha^* g_M \mathcal{H}$. Then F is $\alpha^* g_M$ -irresolute and strongly $\alpha^* g_M$ -closed map. Let S be a $\alpha^* g_M$ -open in U . Then $U-S$ is a $\alpha^* g_M$ -closed in U . Since F is strongly $\alpha^* g_M$ -closed map, $F(U-S)$ is a $\alpha^* g_M$ -closed in V . That is $F(U-S)=V-F(S)$ is a $\alpha^* g_M$ -closed in V implies $F(S)$ is a $\alpha^* g_M$ -open in V . Hence F is strongly $\alpha^* g_M$ -open map. **Conversely**, let $F: U \rightarrow V$ be a strongly $\alpha^* g_M$ -open map and $\alpha^* g_M$ -irresolute. Let S be a $\alpha^* g_M$ -closed in U . Then $U-S$ is a $\alpha^* g_M$ -open in U . Since F is strongly $\alpha^* g_M$ -open, $F(U-S)$ is a $\alpha^* g_M$ -open in V . That is $F(U-S)=V-F(S)$ is a $\alpha^* g_M$ -open in V . Thus $F(S)$ is $\alpha^* g_M$ -closed in V . Then F is strongly $\alpha^* g_M$ -closed map. Hence F is $\mathcal{S}_M \alpha^* g_M \mathcal{H}$.

Theorem: 5.6 Let $(U, \tau_{\mathcal{R}}(X), \mu_{\mathcal{R}}(X))$ and $(V, \tau_{\mathcal{R}}^*(M), \mu_{\mathcal{R}}^*(M))$ be two Micro Topological Spaces. Let $F: U \rightarrow V$ be a $\mathcal{S}_M \alpha^* g_M \mathcal{H}$. Then F is $\mathcal{M-S}_M \mathcal{H}$ whenever U and V are $\alpha^* g_M \mathcal{T}_{1/2}$ -space.

Proof: Let S be a $\alpha^* g_M$ -closed in V . Since F is $\alpha^* g_M$ -irresolute, $F^{-1}(S)$ is $\alpha^* g_M$ -closed in U . Since U is $\alpha^* g_M \mathcal{T}_{1/2}$ -space, $F^{-1}(S)$ is Micro closed in U . Hence F is Micro irresolute. Let T be a $\alpha^* g_M$ -closed in U . Since F is strongly $\alpha^* g_M$ -closed map, $F(T)$ is a $\alpha^* g_M$ -closed in V . Since V is $\alpha^* g_M \mathcal{T}_{1/2}$ -space, $F(T)$ is Micro closed in V . Hence F is strongly Micro closed map. Therefore F is $\mathcal{M-S}_M \mathcal{H}$.

VI. CONCLUSION

In this investigation, we looked into a few characteristics of Micro α^* -Generalized Closed Sets in Micro Topological Spaces. Also we introduced a strongly $\alpha^* g_M$ -homeomorphism, strongly $\alpha^* g_M$ -closed map and strongly $\alpha^* g_M$ -open map. **Applications:** Micro Topology can be applied in analyze the properties of graphs and the relationships between vertices, potentially leading to applications in network analysis and social network theory. Micro Topology finds application in various real world problems, particularly in areas like environmental analysis, medical applications and decision making. It may be analyzed the relationships between different factors and predict outcomes in situations where traditional methods may fall short. Future research in Micro Topology holds significant potential in various fields, including data analysis, computer science and material science.

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